$$\frac{dx_2}{dt} = \frac{q\left(1 - \frac{1}{e^{t/T}}\right)}{T} - \frac{x_2}{T}. \quad \text{Hence, } x_2 = q\left(1 - \frac{1 + \frac{1}{1!}\frac{t}{T}}{e^{t/T}}\right).$$

Similarly,

$$x_3 = q\left(1 - \frac{1 + \frac{1}{1!}\frac{t}{T} + \frac{1}{2!}\left(\frac{t}{T}\right)^2}{e^{t/T}}\right);$$

and so on.

For the *final* amounts in the successive cups, we have

$$X_1 = q\left(1 - \frac{1}{e}\right), \quad X_2 = q\left(1 - \frac{1 + \frac{1}{1!}}{e}\right), \quad X_3 = q\left(1 - \frac{1 + \frac{1}{1!} + \frac{1}{2!}}{e}\right), \quad \cdots.$$

In general, we have

$$X_K = q\left(1 - \frac{e_K}{e}\right)$$

where e_K is the sum of the first K terms of

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \text{etc.}$$

As $K \doteq \infty$, $X_K = 0$.

Note.—Here also the rate of flow is not essential. If we take as the independent variable the amount of wine x which has been poured into the first cup, then the differential equations are

$$\frac{dx_1}{dx} + \frac{x_1}{q} = 1, \qquad \frac{dx_k}{dx} + \frac{x_k}{q} = \frac{x_{k-1}}{q}$$

and

$$x_k = q\left[1 - \left(1 + \frac{x}{q} + \frac{1}{2!}\left(\frac{x}{q}\right)^2 + \cdots + \frac{1}{(k-1)!}\left(\frac{x}{q}\right)^{k-1}\right)e^{-x/q}\right].$$

The final result is obtained by setting $x = q$.—EDITORS.

Also solved by W. D. CAIRNS, ALEXANDER KNISELY, L. C. MATHEWSON, and ARTHUR PELLETIER.

2792 [1919, 414]. Proposed by B. J. BROWN, Kansas City.
Solve the differential equation,

$$x^2(1 - x)\frac{d^2y}{dx^2} + 2x(2 - x)\frac{dy}{dx} + 2(1 + x)y = x^2.$$

SOLUTION BY C. P. SOUSLEY, Pennsylvania State College.

This equation is exact and the first integral is,

$$x^2(1 - x)\frac{dy}{dx} + x(x + 2)y = \frac{x^3 + C}{3},$$

or

$$\frac{dy}{dx} + \frac{x + 2}{x(1 - x)}y = \frac{x^3 + C}{3x^2(1 - x)}.$$

Multiplying through by the integrating factor, $x^2/(1 - x)^3$, we have

$$\frac{x^2}{(1 - x)^3}\frac{dy}{dx} + \frac{x(x + 2)}{(1 - x)^4}y = \frac{x^3 + C}{3(1 - x)^4},$$

and on integrating, we have

$$\frac{x^2}{(1-x)^3}y = \frac{C+1}{9(1-x)^3} - \frac{1}{2(1-x)^2} + \frac{1}{(1-x)} + \log K.\sqrt[3]{(1-x)}.$$

Solved similarly by C. A. Isaacs, Gertrude McCain, and H. L. Olson.

2793 [1919, 458]. Proposed by J. L. Riley, Stephenville, Texas.

If a, b, and c, are complex, and α, β, and γ, real constants, the point

$$x = \frac{at^2 + 2bt + c}{\alpha t^2 + 2\beta t + \gamma}$$

traces a conic or a straight line when t takes all real values.

Discussion by A. F. Frumveller, Marquette University.

Since $\dot{x}$ is a complex number, let us put $x = u + iv$, $a = a_0 + a_1 i$, $b = b_0 + b_1 i$, $c = c_0 + c_1 i$, and clear of fractions. Separating the real and imaginary parts of this equation, we obtain the simultaneous set

(1) $\begin{cases} t^2(\alpha u - a_0) + 2t(\beta u - b_0) + (\gamma u - c_0) = 0, \\ t^2(\alpha v - a_1) + 2t(\beta v - b_1) + (\gamma v - c_1) = 0. \end{cases}$

The eliminant is $|p_0 q_1| \cdot |p_1 q_2| - |p_0 q_2|^2 = 0$ (L. E. Dickson, *Elementary Theory of Equations*, New York, 1914, p. 155), where

$$|p_0 q_1| = 2\begin{vmatrix} \alpha u - a_0 & \beta u - b_0 \\ \alpha v - a_1 & \beta v - b_1 \end{vmatrix}$$

$= 2[(\alpha b_0 - a_0\beta)v + (a_1\beta - \alpha b_1)u + (a_0 b_1 - a_1 b_0)]$ with similar expressions for the other two determinants.

The eliminant is, therefore, a quadratic in (u, v), i.e., a conic, which under suitable conditions degenerates into straight lines. This conic in the plane uov (the plane of the complex number x) is in reality the projection of the actual path of the moving point in space as it spirals its way around the axis of t or a parallel line standing out at right angles to the lines $\overline{ou}$, $\overline{ov}$, in the x-plane.

Cf. an article on "The graph of $f(x)$ for complex numbers" (this Monthly, *1917*, 409), where many analogous examples are worked out and graphed in this rather unusual system of coördinates.

Also solved by Arthur Pelletier.

2796 [1919, 458]. Proposed by N. P. Pandya, Amreli, India.

Construct a triangle ABC having its centroid on a given ellipse, AB being a fixed diameter of the ellipse and C lying on one of the directrices.

Solution by Grace M. Bareis, Ohio State University.

Let O be the center of the given ellipse. Construct OM perpendicular to a directrix and meeting it at M. Determine P on OM so that $OP = \frac{1}{3}OM$. Through P draw a line parallel to the directrix and cutting the ellipse in N_1 and N_2. Draw ON_1 and ON_2 meeting the directrix in C_1 and C_2, respectively. Then ABC_1 or ABC_2 is a solution. It is to be noted that the points C_1 and C_2 are fixed points whatever diameter AB may have been chosen. The problem has four solutions, two corresponding to each directrix, if $e > \frac{1}{3}$; two solutions, one corresponding to each directrix, if $e = \frac{1}{3}$; no real solution when $e < \frac{1}{3}$.

Also solved by E. J. Oglesby, H. L. Olson, and Arthur Pelletier.

2797 [1919, 458]. Proposed by E. J. Oglesby, New York University.

Solve for x and y, the simultaneous equations,

$$x^3 + y^3 = 35 \quad \text{and} \quad x^2 + y^2 = 13.$$